

NJL model with vector couplings vs. phenomenology

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We study the pseudoscalar, vector and axial current correlation functions in SU(2)-NJL model with scalar and vector couplings. The correlation functions are evaluated in leading order in number of colors N_c . As it is expected in the pseudoscalar channel pions appear as Goldstone bosons, and after fixing the cutoff to reproduce the physical pion decay constant, we obtain well-known current-algebra results. For the vector and axial channels we use essentially that at spacelike momenta the correlation functions can be related to the experimentally known spectral density via dispersion relations. We show that the latter imposes strong bounds on the strength of the vector coupling in the model. We find that the commonly used on-shell treatment of the vector and axial mesons (identified as poles at large timelike momenta) fails to reproduce the behavior of the corresponding correlation functions at small spacelike momenta extracted from the physical spectral density. The parameters of the NJL model fixed by the correlation functions at small spacelike momenta differ noticeably from those of the on-shell treatment.

PACS number(s): 12.39.Fe, 11.15.Pg, 11.55.Fv

The correlation functions of different colorless hadronic currents built of quarks and gluons offer a general tool to study the structure of the QCD vacuum and the hadron spectrum [1]. One can obtain information about the physical intermediate states from the pole structure of the correlation functions in the timelike region as well as from its behavior at spacelike momenta. For an exact theory both ways should lead to identical results. This, however, is not necessarily true for the effective models, which in the present lack of non-perturbative solution to QCD attract an increasing interest. Among them, the Nambu – Jona-Lasinio model, proposed [2] in early sixties in analogy with the BCS theory of superconductivity, plays an essential role. This model incorporates the basic symmetries of QCD and in particular, the chiral symmetry. It is able to manifest the spontaneous chiral symmetry breaking and, as a natural consequence, the appearance of the pseudoscalar mesons as Goldstone particles. As such it offers a simple but quite successful working scheme to study the role of the dynamical chiral symmetry breaking mechanism in hadron physics (for review see refs. [4–7]). However, the model suffers from two important drawbacks. First, it is not renormalizable and to make the theory finite one needs a finite cut off fixed in a consistent way (usually to reproduce the pion decay constant) and the limit of large N_c (number of colors). The typical value of the cut off is about 1 GeV. The second problem is the lack of confinement. Obviously, these drawbacks make the so far used on-shell treatment of the (axial) vector mesons questionable [8–10,12,13]. In the present work we will show that the proper way to overcome this problem is to consider the corresponding current correlation function at small spacelike momenta. In this case we do not face the problem of the strong coupling to the $\bar{q}q$ continuum and we apply the model at small momentum region where it is designed for. To this end we make essentially use of the fact that the correlator at small spacelike momenta can be related via dispersion relations to well-established rich phenomenology without detailed knowledge of the dynamics at large timelike momenta. We will show that such a scheme allows to describe the vector modes in the meson sector of the NJL model in a consistent way.

The simplest SU(2) Nambu–Jona-Lasinio (NJL) model is defined by lagrangean [2]:

$$\begin{aligned} \mathcal{L}_{NJL} = & \bar{\Psi} (i\partial - m_0) \Psi + \frac{G_s}{2} [(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\vec{\tau}\gamma_5\Psi)^2] \\ & - \frac{G_v}{2} [(\bar{\Psi}\gamma_\mu\vec{\tau}\Psi)^2 + (\bar{\Psi}\vec{\tau}\gamma_\mu\gamma_5\Psi)^2], \end{aligned} \quad (1)$$

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which contains chirally invariant local four-fermion interactions with scalar G_s and vector G_v couplings¹. The current quark mass m_0 stands for both *up* and *down* quarks (assumed to be degenerated in mass).

Applying the well-known bosonization procedure [14] we arrive at the model generating functional expressed as a functional integral over auxiliary boson fields $S, \vec{P}, \vec{V}_\mu, \vec{A}_\mu$:

$$Z_{NJL} = \int \mathcal{D}S \mathcal{D}\vec{P} \mathcal{D}\vec{V}_\mu \mathcal{D}\vec{A}_\mu e^{N_c \text{Tr} \log \left(D[S, \vec{P}, \vec{V}_\mu, \vec{A}_\mu] \right) + I_m[S, \vec{P}, \vec{V}_\mu, \vec{A}_\mu]}, \quad (2)$$

where the quarks are already integrated out. To simplify our notations we adopt for P , V_μ and A_μ that $P \equiv \vec{P} \cdot \vec{\tau}$ etc. The euclidean Dirac operator D is given by

$$D = -i\partial_\mu \gamma_\mu + S + i P \gamma_5 + V_\mu \gamma_\mu + A_\mu \gamma_5 \gamma_\mu \quad (3)$$

and the trace over colors is explicitly done. The symmetry breaking term is included in the boson part:

$$I_m = \int d^4x \left\{ \frac{1}{2G_s} (S^2 + \vec{P}^2) + \frac{1}{2G_v} (\vec{V}_\mu^2 + \vec{A}_\mu^2) - \frac{m_0}{G_s} S \right\}. \quad (4)$$

In the large N_c limit the integral in eq.(2) is given by its saddle point value. For the translational invariant case (vacuum) we have a non-trivial stationary meson configuration given by

$$S = M, \quad \vec{P} = 0, \quad \vec{V}_\mu = 0, \quad \vec{A}_\mu = 0, \quad (5)$$

which means that the chiral symmetry is spontaneously broken. The quarks acquire a constituent mass M which is related to the scalar coupling constant G_s via the stationary condition (saddle point):

$$M = 2G_s M N_c \text{Tr} \left(\frac{1}{D[M]} \right) + m_0 = m_0 - G_s < \bar{\Psi} \Psi >. \quad (6)$$

Here $< \bar{\Psi} \Psi >$ is the chiral quark condensate. As it was mentioned the model is not renormalizable and in order to make the fermion determinant finite it must be regularized. Thus, the model has four parameters, namely two coupling constants G_s and G_v , the current mass m_0 and the cut off Λ . Usually they are fixed reproducing the physical pion decay constant and the physical masses of pion and rho meson. It leaves one parameter free which is commonly chosen to be the constituent mass M because with eq.(6) one can eliminate G_s in favor of M . In principle, one can use the empirical value for the quark condensate $< \bar{\Psi} \Psi >$ to constrain M . However, being quadratically divergent $< \bar{\Psi} \Psi >$ is very sensitive to the details of the regularization scheme.

In this paper we suggest an alternative way of fixing the model parameters G_s , G_v , m_0 and Λ . To this end we study the correlation functions of the pseudoscalar, vector and axial-vector currents:

$$P^a(x) = \bar{\Psi}(x) i \gamma_5 \frac{\tau^a}{2} \Psi, \quad (7)$$

$$V_\mu^a(x) = \bar{\Psi}(x) \gamma_\mu \frac{\tau^a}{2} \Psi, \quad (8)$$

$$A_\mu^a(x) = \bar{\Psi}(x) \gamma_\mu \gamma_5 \frac{\tau^a}{2} \Psi, \quad (9)$$

in the NJL model. The correlation functions are defined by the vacuum-to-vacuum matrix elements of the currents and can be expressed in terms of invariant functions as follows:

$$\Pi_P(Q^2) = \int d_4x e^{iqx} \langle 0 | \mathcal{T} \{ P^a(x) P^a(0) \} | 0 \rangle, \quad (10)$$

¹This lagrangean with vector couplings was considered first by Kikkawa [3]

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_v(Q^2) = \int d_4x e^{iqx} \langle 0 | \mathcal{T} \{ V_\mu^a(x) V_\mu^a(0) \} | 0 \rangle, \quad (11)$$

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_a^t(Q^2) + q_\mu q_\nu \Pi_a^l(Q^2) = \int d_4x e^{iqx} \langle 0 | \mathcal{T} \{ A_\mu^a(x) A_\mu^a(0) \} | 0 \rangle, \quad (12)$$

where as usual Q is defined as $Q^2 = -q^2$. In contrast to the vector current in the real world the axial-vector current is not conserved and the correlation function in the axial channel has an additional longitudinal part $\Pi_a^l(Q^2)$ which vanishes in the chiral limit $m_0 \rightarrow 0$. In the NJL model² the matrix element of rhs of eqs.(10-12) can be expressed as a path integral using the model generating functional (2). In the leading order in N_c only two diagrams (shown in fig. 1) contribute to the matrix elements. The first one is a simple one-quark loop whereas the second includes a boson line. For the pseudoscalar function we get the following result:

$$\Pi_a(Q^2) = \frac{Q^2 Z_p(Q^2) f_a(Q^2) - \frac{\langle \bar{\Psi} \Psi \rangle}{M}}{G_s f_a(Q^2) Z_p(Q^2)} \frac{1}{Q^2 + \frac{m_0}{M G_s} \frac{1}{B_p(Q^2) f_a(Q^2)}}, \quad (13)$$

where

$$f_a(Q^2) = \frac{1}{1 + 4M^2 G_v Z_p(Q^2)}. \quad (14)$$

Function $Z_p(Q^2)$ corresponds to the quark-loop diagram with two pseudoscalar-isovector vertices ($i\gamma_5 \tau_a$). As already mentioned, the model is not renormalizable and in order to make the quark loop finite we need a regularization. In our calculations we use the proper-time as well as the Pauli-Villars regularization scheme. Both schemes preserve the symmetries of the model. Here we present only the proper-time regularized expression for $Z_p(Q^2)$ [12]:

$$Z_p(Q^2) = \frac{4N_c}{(2\pi)^4} \int_{-1}^1 \frac{du}{2} \int_{\Lambda^{-2}}^{\infty} \frac{ds}{s} e^{-s} \left[M^2 + \frac{Q^2}{4} (1-u^2) \right], \quad (15)$$

and for the condensate:

$$\langle \bar{\Psi} \Psi \rangle = -\frac{N_c}{2\pi^2} M \int_{\Lambda^{-2}}^{\infty} \frac{ds}{s^2} e^{-sM^2}. \quad (16)$$

Details as well as regularized expressions for the quark condensate and the function $Z_p(Q^2)$ in the case of Pauli-Villars regularization can be found in ref. [15].

In the chiral limit the pseudoscalar correlation function (13) develops the expected Goldstone pole at $Q^2 = 0$:

$$\Pi_a(Q^2 \rightarrow 0) \longrightarrow \frac{\langle \bar{\Psi} \Psi \rangle^2}{Q^2 f_\pi^2} \quad (17)$$

where the pion decay constant f_π^2 is given by the residue of the pole:

$$f_\pi^2 = M^2 f_a(0) Z_p(0). \quad (18)$$

The non-zero current mass m_0 shifts the pion pole in eq.(17) to $Q^2 = -m_\pi^2$ and one recovers the Gell-Mann - Oakes - Renner relation:

$$f_\pi^2 m_\pi^2 = -m_0 \langle \bar{\Psi} \Psi \rangle + O(m_0^2). \quad (19)$$

In the vector channel the correlation function $\Pi_v(Q^2)$ is given by

$$\Pi_v(Q^2) = \frac{1}{4} \frac{Z_v(Q^2)}{1 + G_v Z_v(Q^2) Q^2}, \quad (20)$$

²Using different techniques the invariant functions are evaluated also in ref. [13]

where, similarly to $Z_p(Q^2)$, the function $Z_v(Q^2)$ corresponds to the quark-loop diagram with two vector vertices ($\gamma_\mu \tau^a$) (lhs of fig. 1). In particular, in the proper-time scheme it has the following form:

$$Z_v(Q^2) = \frac{4N_C}{(2\pi)^4} \int_{-1}^{+1} \frac{du}{2} (1-u^2) \int_{\Lambda^{-2}}^{\infty} \frac{ds}{s} e^{-s \left[M^2 + \frac{Q^2}{4} (1-u^2) \right]}. \quad (21)$$

For the transverse invariant function of the axial current correlator in the NJL model we have:

$$Q^2 \Pi_a^t(Q^2) = \frac{1}{4} \frac{Z_v(Q^2) Q^2 + 4M^2 Z_p(Q^2)}{1 + G_v Z_v(Q^2) Q^2 + 4G_v M^2 Z_p(Q^2)}, \quad (22)$$

whereas the longitudinal one includes only a pion contribution:

$$Q^2 \Pi_a^l(Q^2) = - \frac{f_\pi^2 m_\pi^2}{Q^2 + \frac{m_\pi^2 f_\pi^2}{M^2 Z_p(Q^2) f_a(Q^2)}}. \quad (23)$$

The correlation functions (20),(22) possess (at some model parameter values) poles in the timelike region $Q^2 < 0$ which is in fact the pole of the propagator of the auxiliary (axial) vector field. This is used in the so-called on-shell treatment [8–13] where one fixes the vector coupling constant G_v adjusting the position of the pole to the physical ρ mass. The other model parameters, namely the cutoff Λ and the current mass m_0 are fixed reproducing the physical pion mass and the physical pion decay constant from eqs. (18),(19), whereas the constituent quark mass is treated as a free parameter as usual. To illustrate the on-shell scheme we present in fig.2 the corresponding parameter values (index “pole”) as a function of M for the case of the Pauli-Villars regularization. Since the results with the proper-time regularization are almost identical with those of the proper-time ones we do not discuss them separately. As can be seen the behavior of the coupling constants and the cutoff as function of the constituent mass is not smooth – the value $M = m_\rho^{exp}/2$ is a singular point where all curves show a kink. At $M > m_\rho^{exp}/2$ the vector current correlation function has a real pole at $Q^2 = -m_\rho^2$ which is an indication for a bound state in the spectrum. However, because of lack of confinement a second peak in the $\bar{q}q$ continuum appears, which is not taken into account in the definition of the physical ρ field. Obviously, it is hard to relate the second peak to the excited $\rho(1450)$ -resonance. At smaller values of $M < m_\rho^{exp}/2$ in the vector channel a broad resonance appears in the $\bar{q}q$ continuum because the corresponding pole moves to the complex plane of Q^2 . The position of the peak depends on the vector coupling constant G_v . Using this³ we determine G_v fixing the position of the peak in the vector spectral function $\text{Im}\Pi_v$ at the physical ρ mass. However, as can be seen from fig.2 it is not always possible to find a solution for G_v : for M between 230 and 260 MeV in fact there is no such a solution. It should be also noted that at $M > m_\rho^{exp}/2$ both G_v and G_s , as well as the cutoff Λ show a stronger dependence on M as G_v is dominant ($G_v > G_s$). This changes for smaller values of M where G_s and Λ stay almost constant whereas G_v decreases. In the axial channel for the values of the constituent mass M considered (200 - 450 MeV) the A_1 meson appears as a very broad peak being centered around 1 GeV in the $\bar{q}q$ continuum. In Fig.2 we also show the parameter values (index “grad”) obtained by means of a gradient or heat-kernel expansion of the effective action [16,17]. This procedure is frequently used to fix the vector coupling constant G_v in NJL model and in fact it is an approximation to the on-shell description of vector modes. As can be seen from Fig.2 this procedure provides a quite crude approximation to the on-shell values because of the large masses of the vector mesons. From the above discussion one concludes that the on-shell treatment is not able to provide a consistent description of the vector meson modes in NJL model.

As a next step we suggest to make use of the fact that at spacelike momenta $Q^2 > 0$ the vector and axial-vector current correlation functions can be related to the experimentally known spectral density via dispersion relations. In the vector channel the invariant function $\Pi_v(Q^2)$ satisfies a dispersion relation with one subtraction:

$$\Pi_v(Q^2) = \Pi_v(0) - \frac{Q^2}{\pi} \int ds \frac{\text{Im}\Pi_v(s)}{s(s+Q^2)}, \quad (24)$$

where the physical spectral density $\text{Im}\Pi_v(s)$ is experimentally accessible:

$$\text{Im}\Pi_v(s) = \frac{1}{12\pi} \frac{\sigma_{e^+e^- \rightarrow h}(s)}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}(s)}. \quad (25)$$

³Such a procedure was used in refs. [9,10]

For the rhs we use a simple (ρ + continuum) parameterization [18] of the experimental data

$$\text{Im}\Pi_v(s) = \frac{\pi m_\rho^2}{g_\rho^2} \delta(s - m_\rho^2) + \frac{1}{8\pi} \left(1 + \frac{\alpha_s}{\pi}\right) \Theta(s - s_0), \quad (26)$$

with the parameter values:

$$m_\rho^2 = 0.77 \text{ GeV}^2, \quad \left(\frac{g_\rho^2}{4\pi}\right) = 2.36, \quad s_0 = 1.5 \text{ GeV}^2. \quad (27)$$

Eqs.(24) and (26) provide an independent way to fix the vector coupling constant G_v confronting the model with the experiment by equating the phenomenological dispersion integral and the model correlation function at low positive Q^2 :

$$\Pi_v(0) - \frac{Q^2}{\pi} \int ds \frac{\text{Im}\Pi_v(s)}{s(s+Q^2)} = \frac{1}{4} \frac{Z_v(Q^2)}{1 + G_v Z_v(Q^2) Q^2}. \quad (28)$$

The resulting G_v as well as Λ and G_s as a function of M , obtained by adjusting (χ^2 -fit) the lhs of eq. (28) to the rhs with the physical spectral density (26) at low positive momenta (we consider $0 < Q^2 < 0.5 \text{ GeV}^2$), are shown (index “corr”) in fig.2 a) and b). Apparently the correlation function at spacelike momenta and the on-shell treatment yield rather different results. In contrast to the curves resulting from the on-shell scheme those of the correlation function have the advantage to behave smoothly as a function of M . This, and the very fact that the NJL model is designed for low-energy structures involving small momenta, makes the above correlation method preferable. With increasing M larger values for G_v are needed to fit the phenomenological side whereas the G_s is slightly decreasing.

In the axial channel, since the longitudinal part of the axial correlation function is almost independent on the G_v and M , we will concentrate our further discussion on its transverse part Π_a^t . It obeys a dispersion relation with two subtractions:

$$Q^2 \Pi_a^t(Q^2) = \Pi_a^t(0) + Q^2 \frac{d\Pi_a^t(0)}{dQ^2} + \frac{q^4}{\pi} \int ds \frac{\text{Im}\Pi_a^t(s)}{s^2(s+Q^2)}. \quad (29)$$

Similar to the vector channel for $\text{Im}\Pi_a^t(Q^2)$ we take a simplified (A_1 + continuum) parameterization [18]

$$\text{Im}\Pi_a^t(s) = \frac{\pi m_{A_1}^4}{g_{A_1}^2} \delta(s - m_{A_1}^2) + \frac{1}{8\pi} \left(1 + \frac{\alpha_s}{\pi}\right) \Theta(s - s_0), \quad (30)$$

with parameters [19]:

$$m_{A_1}^2 = 1.26 \text{ GeV}^2, \quad \left(\frac{4\pi}{g_{A_1}^2}\right) = 0.15 - 0.18, \quad s_0 = 1.7 \text{ GeV}^2. \quad (31)$$

Confronting the model A_1 -correlation function (22) and the phenomenological expressions (29), (30) one has another way to fix the vector coupling constant G_v . The obtained vector coupling constant ($G_{v(a_1)}^{\text{corr}}$) as a function of M is shown in fig.2 a) in comparison with the results from fitting the ρ phenomenology as well as from the on-shell ρ treatment and the derivative expansion. In contrast to the vector channel the model axial function $\Pi_a^t(Q^2)$ depends much stronger on the constituent mass and because of that the model is able to fit simultaneously both ρ - and A_1 -channels only within a narrow window of values for the constituent quark mass M around 240 MeV. It is interesting to note that in this window the on-shell values are also very close to those obtained from the correlation functions. The particular value of M depends on the parameter values (27),(31) used for the (resonance + continuum) parameterizations. In fact the estimate of the constituent mass $M \approx 300 \text{ MeV}$ in the random instanton model [21,22] of QCD vacuum, applied successfully also to the meson correlation functions [23], is not far from our numbers. The resulting vector coupling constant G_v is of the same order as the scalar one G_s . In this point we disagree with ref. [20] in which constraining the NJL model via QCD sum rules it is concluded that G_v should be almost zero or at least $G_v \ll G_s$. It is due to the fact that in contrast to us in ref. [20] the NJL model is treated in a quite simplified approximate way ignoring the full Q^2 -dependence of the correlation functions as well as the relations between the model parameters fixed to reproduce the physical pion properties.

In our calculations so far we used simple parameterizations (26),(30) for the experimental spectral densities in which the resonance widths are neglected. In order to check this approximation we repeated the calculations using a much more elaborated (finite-width resonance+continuum) parameterization [24] of the experimental data. The obtained results follow qualitatively the picture of fig. 2. The only difference is that the narrow window, where in

both vector and axial channels the experimental low-energy behavior of the correlation functions can be reproduced simultaneously, is shifted to higher values of the constituent mass $M \approx 260$ MeV.

We also related the model with G_v , fixed in the present scheme, to the chiral effective lagrangean of Gasser and Leutwyler [26]. Following ref. [25] we calculated the corresponding low-energy coefficients $\bar{l}_1 - \bar{l}_6$ in the notation of ref. [26] and low-energy pionic characteristics. As can be seen from Table I our theoretical predictions are in good agreement with the empirical values.

To summarize, in this letter we study the vector meson modes in the NJL model in terms of current correlation functions. We find that the on-shell treatment of the vector meson fails to reproduce the low-energy behavior of the correlation functions fixed by the experimentally known spectral density via dispersion relations. We offer a different scheme to fix the NJL-model parameters based on the behavior of the correlation functions at small spacelike momenta. Treating the vector coupling constant as a free parameter the model is able to reproduce the phenomenology in both the axial- and vector channels for the constituent mass only in a narrow window around 240 MeV. The vector coupling constant is by no means zero and is of the same order as the scalar coupling constant. In principle, these two schemes, the on-shell treatment (large timelike momenta) and the one based on the behavior of the correlation function at small spacelike momenta, should provide identical results within an “exact” theory, which however is not the case for an effective model like NJL. In the latter case the correlation function method appears to be preferable. One also should keep in mind that the present considerations are done in leading order in N_c . The inclusion of $1/N_c$ quantum boson (loop) corrections could in principle change the present results.

ACKNOWLEDGEMENT

We would like to thank Dmitri Diakonov, Victor Petrov and Georges Ripka for helpful discussions. The project has been partially supported by the VW Stiftung, DFG and COSY (Jülich).

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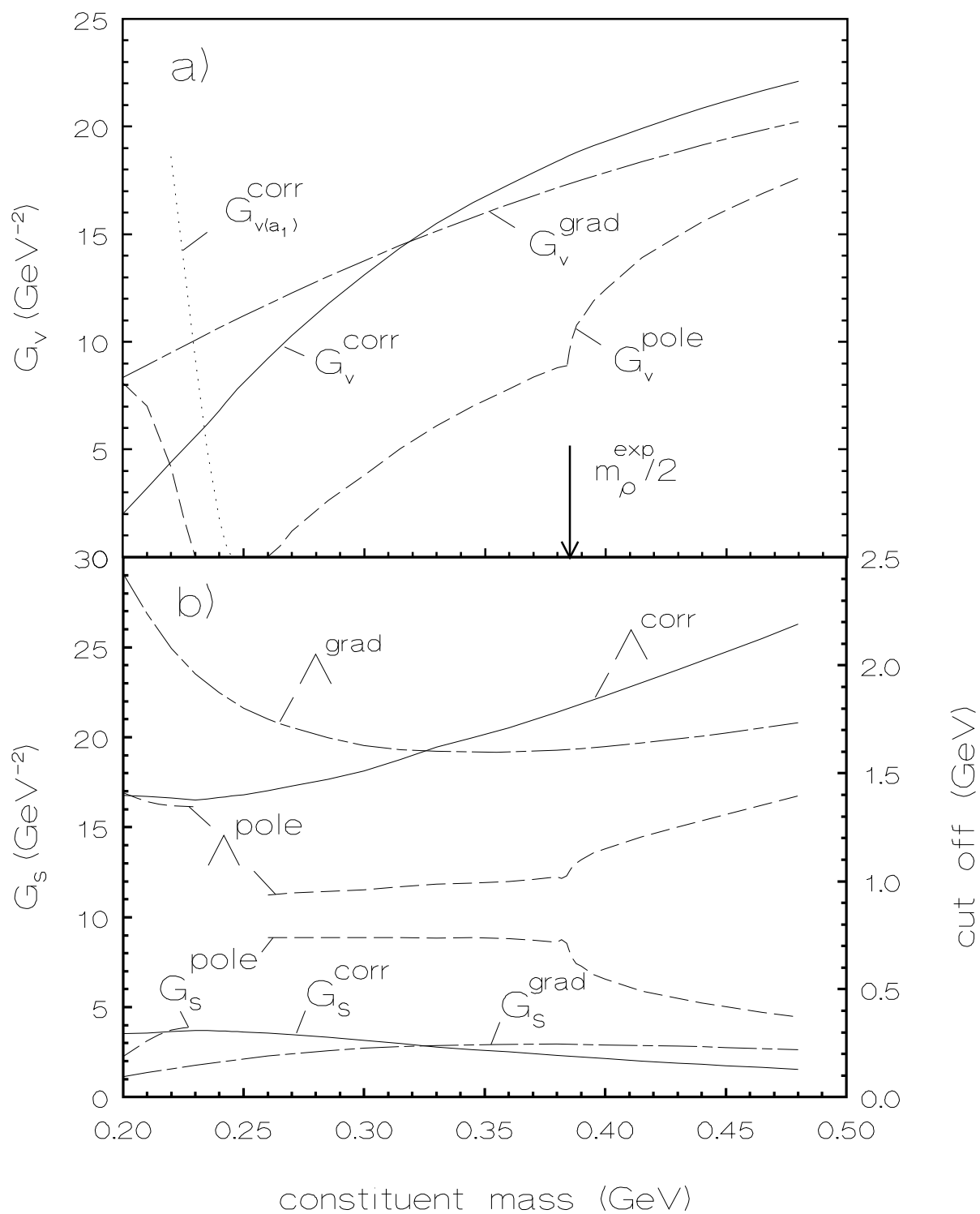
FIG. 1. Diagrams contributing to the correlation functions in leading N_c order.

FIG. 2. The parameters of the model as function of the constituent mass M : a) vector coupling constant G_v fixed in the on-shell treatment G_v^{pole} , from derivative expansion G_v^{grad} , and from the correlation functions in the case of ρ - and a_1 -phenomenology fits; b) the same as a) but for the scalar coupling constant G_s and the cut off Λ .

TABLE I. Low-energy SU(2) coefficients and low-energy pion properties for constituent mass $M = 235$ MeV.

SU(2) coefficients	model	phenom.	π properties	model	exp.
Λ [MeV]	1400		a_0^0	0.16	0.26 ± 0.05
$-\langle \bar{q}q \rangle^{1/3}$ [MeV]	310	265 ± 40	b_0^0	0.19	0.25 ± 0.03
m_0 [MeV]	3.5	7 ± 3	a_0^2	-0.045	-0.028 ± 0.012
\bar{l}_1	-4.17	-4.56 ± 1.5	b_0^2	-0.084	-0.082 ± 0.008
\bar{l}_2	3.02	2.3 ± 0.7	a_1^1	0.035	0.038 ± 0.002
\bar{l}_3	2.45	-1.0 ± 3	$a_2^0 \times 10^4$	9.8	17.0 ± 3.0
\bar{l}_4	2.44	1.1 ± 0.4	$a_2^2 \times 10^4$	-1.1	1.0 ± 3.0
\bar{l}_5	9.1	10.4 ± 1.3	$\langle r^2 \rangle_s^\pi [\text{fm}^2]$	0.43	0.52 ± 0.15
\bar{l}_6	12.7	13.1 ± 1.3	$\langle r^2 \rangle_v^\pi [\text{fm}^2]$	0.37	0.44 ± 0.03

$$\begin{array}{c}
 \begin{array}{c} \gamma_\mu \\ \times \\ \gamma_\mu \gamma_5 \\ \gamma_5 \tau_a \end{array}
 \quad
 \begin{array}{c} \gamma_\nu \\ \times \\ \gamma_\nu \gamma_5 \\ \gamma_5 \tau_a \end{array}
 \quad
 +
 \quad
 \begin{array}{c} \gamma_\mu \\ \times \\ \gamma_\mu \gamma_5 \\ \gamma_5 \tau_a \end{array}
 \quad
 \begin{array}{c} \gamma_\nu \\ \times \\ \gamma_\nu \gamma_5 \\ \gamma_5 \tau_a \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \rho \\
 \text{---} \\
 A, \pi
 \end{array}
 \quad
 \begin{array}{c}
 \gamma_\nu \\
 \times \\
 \gamma_\nu \gamma_5 \\
 \gamma_5 \tau_a
 \end{array}$$



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